

# Modern Exterior Ballistics



The Launch  
and Flight

Dynamics of  
Symmetric  
Projectiles

Robert L. McCoy

# Contents

Preface .....	9
<b>Chapter 1</b> A Brief History of Exterior Ballistics .....	10
1.1 Introduction .....	10
1.2 Early Beginnings .....	10
1.3 Exterior Ballistics in the Nineteenth Century .....	10
1.4 Early Twentieth Century Developments .....	13
1.5 The First Modern Aerodynamic Force-Moment System for Projectiles .....	17
1.6 The Beginnings of Computational Aerodynamics .....	17
1.7 Exterior Ballistics Research During the Second World War .....	18
1.8 Post-War Progress in Exterior Ballistics .....	28
1.9 Future Developments .....	29
<b>Chapter 2</b> Aerodynamic Forces and Moments Acting on Projectiles .....	32
2.1 Introduction .....	32
2.2 Drag Force .....	33
2.3 Spin Damping Moment .....	33
2.4 Rolling Moment for Canted Fin Projectiles .....	34
2.5 Lift and Normal Forces .....	34
2.6 Overturning Moment .....	36
2.7 Magnus Force .....	36
2.8 Magnus Moment .....	37
2.9 Centers of Pressure of the Normal Force and the Magnus Force .....	37
2.10 Pitch Damping Force .....	38
2.11 Pitch Damping Moment .....	38
2.12 Neglected Forces and Moments .....	39
2.13 The Effect of Center of Gravity Location on the Aerodynamic Forces and Moments .....	39
2.14 Modern Aeroballistic and Older Ballistic Nomenclatures .....	40
2.15 Summary .....	40
<b>Chapter 3</b> The Vacuum Trajectory .....	42
3.1 Introduction .....	42
3.2 Equations of Motion .....	42
3.3 Discussion of the Vacuum Trajectory .....	44
3.4 Firing Uphill and Downhill .....	47
3.5 Summary .....	51
<b>Chapter 4</b> Notes on Aerodynamic Drag .....	52
4.1 Introduction .....	52
4.2 Classical Drag Measurements .....	52
4.3 The Physical Nature of Drag .....	55
4.4 Airflow Regimes .....	55
4.5 The Effect of Projectile Shape on Drag .....	70
4.6 The Effect of a Burning Tracer on Drag .....	73
4.7 The Effect of Fins on the Drag .....	74
4.8 The Drag of Smooth Spheres .....	76
4.9 The Effect of Yaw on Drag .....	78
4.10 Minimum Drag Projectile Shapes .....	80
4.11 Summary .....	84

<b>Chapter 5</b>	The Flat-Fire Point Mass Trajectory .....	88
5.1	Introduction .....	88
5.2	Equations of Motion .....	89
5.3	The Flat-Fire Approximation .....	90
5.4	Special Analytical Solutions of the Flat-Fire Equations .....	91
5.5	Constant Drag Coefficient .....	92
5.6	Drag Coefficient Inversely Proportional to Mach Number .....	93
5.7	Drag Coefficient Inversely Proportional to the Square Root of Mach Number .....	94
5.8	Comparison of Flat-Fire Trajectory Approximations .....	95
5.9	Summary .....	96
<b>Chapter 6</b>	The Siacci Method for Flat-Fire Trajectories	98
6.1	Introduction .....	98
6.2	Siacci Assumptions and Approximations .....	98
6.3	Derivation of the Siacci Functions .....	98
6.4	The Computation of Siacci Ballistic Tables .....	101
6.5	The Practical Use of the Ballistic Tables .....	101
6.6	Form Factors of Typical Small Arms Projectiles .....	106
6.7	The Effect of Projectile Shape on the Form Factor .....	106
6.8	Rules for the Use of the Form Factor Charts .....	111
6.9	Additional Notes on Form Factors .....	111
<b>Chapter 7</b>	The Effect of Wind on Flat-Fire Trajectories .....	157
7.1	Introduction .....	157
7.2	Equations of Motion .....	157
7.3	The Flat-Fire Approximation .....	158
7.4	The Effect of a Constant Crosswind on the Flat-Fire Trajectory .....	158
7.5	The Effect of a Variable Crosswind on the Flat-Fire Trajectory .....	159
7.6	The Effect of Rangewind on the Flat-Fire Trajectory .....	162
7.7	Summary .....	164
<b>Chapter 8</b>	The Point-Mass Trajectory .....	165
8.1	Introduction .....	165
8.2	Equations of Motion .....	165
8.3	Change of Independent Variable from Time to Distance .....	165
8.4	Numerical Solution of the Equations of Motion .....	166
8.5	Standard Atmospheres for Point-Mass Trajectories .....	166
8.6	Examples of Point-Mass Trajectories .....	169
8.7	Comparison of Point-Mass and Siacci Trajectories .....	172
8.8	The Coriolis Effect on Point-Mass Trajectories .....	178
8.9	Summary .....	183
<b>Chapter 9</b>	Six-Degrees-of-Freedom (6-DOF) and Modified Point-Mass Trajectories .....	187
9.1	Introduction .....	187
9.2	Equations of Motion for Six-Degrees-of-Freedom Trajectories .....	187
9.3	Initial Conditions for Six-Degrees-of-Freedom Trajectories .....	191
9.4	Numerical Solution of Six-Degrees-of-Freedom Trajectories .....	193
9.5	Examples of Six-Degrees-of-Freedom Trajectories .....	194
9.6	Summary and Comments on Six-Degrees-of-Freedom Trajectories .....	212
9.7	The Modified Point-Mass Trajectory Model .....	212
9.8	Examples of Modified Point-Mass Trajectories .....	214

<b>Chapter 10</b>	Linearized Pitching and Yawing Motion of Rotationally Symmetric Projectiles .....	221
10.1	Introduction .....	221
10.2	Equations of Motion for the Linearized Problem .....	221
10.3	Solution of the Differential Equations for Velocity and Spin .....	228
10.4	Simplified Pitching and Yawing Motion of a Spinning Projectile .....	229
10.5	The Classical Gyroscopic Stability Criterion .....	230
10.6	The Yaw of Repose for Spin-Stabilized Projectiles .....	231
10.7	Initial Conditions for Simplified Epicyclic Motion .....	231
10.8	Complete Linearized Pitching and Yawing Motion of Projectiles .....	232
10.9	Gyroscopic and Dynamic Stability of Symmetric Projectiles .....	233
10.10	Initial Conditions for Damped Epicyclic Motion .....	234
10.11	An Example of the Linearized Pitching and Yawing Motion .....	235
10.12	The Motion of the Rotating [ i, j, k ] Coordinate System .....	236
10.13	Pitching and Yawing Motion of a Slightly Asymmetric Missile .....	237
10.14	Summary .....	238
<b>Chapter 11</b>	Linearized Swerving Motion of Rotationally Symmetric Projectiles .....	240
11.1	Introduction .....	240
11.2	The Differential Equation of Swerving Motion .....	240
11.3	Solution of the Differential Equation for Swerve .....	243
11.4	Discussion of the Linearized Swerving Motion .....	244
<b>Chapter 12</b>	Lateral Throwoff and Aerodynamic Jump .....	252
12.1	Introduction .....	252
12.2	Derivation of the Lateral Throwoff Effect .....	254
12.3	The Effect of a Slight Mass Asymmetry on the Initial Pitching and Yawing Motion of a Spinning Projectile .....	255
12.4	The Generalized Aerodynamic Jump Effect .....	259
12.5	The Effect of Mass Asymmetry on Lateral Throwoff and Aerodynamic Jump .....	260
12.6	Derivation of Kent's Equation for a Small Mass Asymmetry .....	264
12.7	The Effect of In-Bore Yaw on Lateral Throwoff and Aerodynamic Jump .....	264
12.8	Derivation of Kent's Equation for a Small In-Bore Yaw .....	266
12.9	The Aerodynamic Jump Due to Crosswind .....	267
12.10	Firing Sidewise From an Airplane .....	270
12.11	Summary .....	272
<b>Chapter 13</b>	Nonlinear Aerodynamic Forces and Moments .....	273
13.1	Introduction .....	273
13.2	Analysis of Nonlinear Drag Coefficient Data .....	273
13.3	Quasi-Linear Analysis of a Cubic Pitching Moment .....	275
13.4	The Effect of a Cubic Pitching Moment on Stability .....	279
13.5	Pitching and Yawing Motion With All Nonlinear Moments .....	280
13.6	Bi-Cubic and Tri-Cubic Magnus Moments .....	284
13.7	Nonlinear Magnus Moments and Limit-Cycle Yawing Motion .....	287
13.8	Quasi-Linear Analysis of a Cubic Lift Force .....	293
<b>Chapter 14</b>	Measurement of Aerodynamic Forces and Moments .....	299
14.1	Introduction .....	299
14.2	Wind Tunnel Methods .....	299
14.3	Free-Flight Ballistic Ranges .....	303
14.4	Classical Data Reduction for Spark Photography Ranges .....	304
14.5	Six-Degrees-of-Freedom Data Reduction for Spark Ranges .....	317
14.6	Modern Data Reduction for Yaw-Card Firings .....	318
14.7	Methods of Yaw Induction .....	320
14.8	Yawsonde Testing	323